Nonlinear Predictive Control for the Four-Tanks Plant Flow Regulation

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Abstract

The four-tanks system is well-known and a considerable number of works study it in the existent literature. This work proposes a nonlinear model predictive control (NMPC) design to regulate the water flow inputs of the four tanks system. As an initial approach, the implementation is performed in Simulink® prior to a foreseeable real-plant application of the proposed solution. Simulation results are presented and discussed to show the performance of the controller. Improvements are enumerated and analysed.

Keywords: predictive control, nonlinear model, zero dynamics, four-tanks system

1 Introduction

The objective of the present work is to design a controller to participate in the CEA challenge, the first phase of which, consists in the design of a controller and test its behaviour in a simulation environment. However, the second phase of the challenge includes the implementation of the proposed solution in a real system. Because of this, certain aspects of the design procedure will have to take into account the applicability of the controller to a real operating plant.

Since MPC is based on the mathematical model of the plant (the four-tanks system in this case) it is a technique very sensitive to model accuracy. This dependence on the model allows several approaches when designing a MPC-based control strategy: from the linear case [7], to the nonlinear MPC (NPMC) [10], hybrid models [8] and piecewise affine (PWA) systems [2].

In this work, to achieve the control objectives, a model based NMPC strategy is proposed to control the nonlinear system. The inclusion of the nonlinearities of the plant in the controller allow to perform the regulation of the studied plant. In recent studies [9] these kind of approaches have been analysed with good results and the computer burden is low enough to consider real-time applications for the controllers.

The main contribution of this paper is a nonlinear implementation of a model predictive control that is sufficiently fast enough to be implemented in the future. The discretization time and the control and prediction horizons are a key aspect to take into account when performing the design and tuning of the controller.

The rest of the paper is organised as follows. To introduce the problem, the four tanks system description can be found in section 2. In section 3 the proposed NMPC strategy theoretical basis is described. Simulation results are extracted and studied in detail in section 4. Finally, in section 5 the conclusions of this paper are presented and some improvements are proposed for the future.

2 System Description

The plant employed to design the controllers is the four-tanks system described by [5]. It is a multivariable system with two manipulable variables (water flows q_a and q_b) and four state variables (water levels h_1, h_2, h_3 and h_4). Depending on the aperture of de valves (determined by the values γ_a and γ_b) the system dynamics present non-minimum phase dynamics and nonlinearities.

The differential equations that model the nonlinear dynamics of the system can be expressed as

$$A\frac{dh_1}{dt} = -a_1\sqrt{2gh_1} + a_3\sqrt{2gh_3} + \gamma_a\frac{q_a}{3600}, \quad (1a)$$

$$A\frac{dh_2}{dt} = -a_2\sqrt{2gh_2} + a_4\sqrt{2gh_4} + \gamma_b\frac{q_b}{3600}, \quad (1b)$$

$$A\frac{dh_3}{dt} = -a_3\sqrt{2gh_3} + (1-\gamma_b)\frac{q_b}{3600}, \quad (1c)$$

$$A\frac{dh_4}{dt} = -a_4\sqrt{2gh_4} + (1-\gamma_a)\frac{q_a}{3600}. \quad (1d)$$

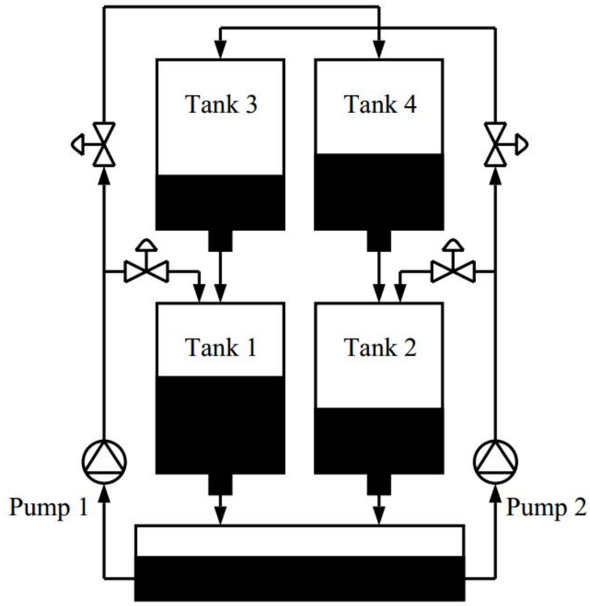

Figure 1: Schematic of the four-tank system [3]

From equation (1) a control-oriented model will be developed to implement the NMPC strategy as described in the next section. A graphical schematic of the four-tanks system is shown in figure 1.

3 Proposed Solution

3.1 The Nonlinear Model Predictive Control Approach

The NMPC strategy is chosen with the objective of not losing accuracy due to the linearisation of the system needed in the traditional MPC approach. The advantage of this formulation is that it is possible to find the optimal control solution of the real system at the cost of changing a convex constrained optimisation to non-convex optimisation. Usually the main drawback is the time needed to find solutions, but given the system's sampling time, the computing time is small enough to let the solver find a solution. Suboptimal solutions are found due to the flatness of the gradient of the cost function, but this solutions are close enough to the optimal solution for the objective of this work and close enough to guarantee feasibility in the next time-step.

To guarantee the feasibility of the next time-stem the imposition of terminal conditions can be added to the control scheme. This addition increases the computational effort to conditions where the solution can not be found inside the sampling range of time, hence it has been discarded from the adopted solution.

3.2 Control-oriented Model

From (1), a control-oriented discretized model is obtained

$$x_1(k+1) = \left(-\frac{a_1}{A}\sqrt{2gx_1(k)}\right)T_s + \left(\frac{a_3}{A}\sqrt{2gx_3(k)} + \frac{\gamma_a}{A}\frac{u_1(k)}{3600}\right)T_s + x_1(k), \tag{2a}$$

$$x_2(k+1) = \left(-\frac{a_2}{A}\sqrt{2gx_2(k)}\right)T_s + \left(\frac{a_4}{A}\sqrt{2gx_4(k)} + \frac{\gamma_b}{A}\frac{u_2(k)}{3600}\right)T_s + x_2(k), \tag{2b}$$

$$x_3(k+1) = \left(-\frac{a_3}{A}\sqrt{2gx_3(k)}\right)T_s + \left(\frac{(1-\gamma_b)}{A}\frac{u_2(k)}{3600}\right)T_s + x_3(k), \tag{2c}$$

$$x_4(k+1) = \left(-\frac{a_4}{A}\sqrt{2gx_4(k)}\right)T_s + \left(\frac{(1-\gamma_a)}{A}\frac{u_1(k)}{3600}\right)T_s + x_4(k), \tag{2d}$$

where the state variables are the h_i tank water levels for $1 \leq i \leq 4$. Moreover, the control inputs are $u_1 \triangleq q_a$ and $u_2 \triangleq q_b$, corresponding to the two input flows to the upper tanks. Letter k determines the discrete-time variable.

3.3 System Constraints

The overall problem constraints are given in the CEA challenge document [1]. The bounding state constraints are defined as

$$0.2\,m \leq x_i \leq 1.2\,m, \quad i = 1,2,3,4. \tag{3}$$

In addition to the state constraints, there is another set of hard constraints related to the input water flows that feed the upper level tanks

$$0\,m^3/h \leq u_z \leq 2.5\,m^3/h, \quad z = a,b. \tag{4}$$

3.4 Cost Function

The objective of the controller is to minimize the function over the control horizon. The function to minimize is $J(h,q,c,p)$, given by the problem statement of the CEA challenge

$$J(h,q,c,p) = (q_a^2 + cq_b^2) + p\frac{V_{min}}{A(h_1+h_2)}. \tag{5}$$

To implement the NMPC algorithm the cost function expressed in (5) has to be discretized and expressed in state representation form

$$J(k) = (u_1(k)^2 + cu_2(k)^2) + p\frac{V_{min}}{A(x_1(k)+x_2(k))}. \tag{6}$$

The cost function is computed over a given prediction and control horizons (H_p and H_c).

3.5 NMPC Algorithm

Let

$$\mathbf{u}(k) \triangleq (u(0|k), \dots, u(H_p - 1|k)) \quad (7)$$

be the sequence of control inputs over a prediction horizon H_p where there is also the dependence on the initial condition $x(0|k) \triangleq x_0$. The NMPC algorithm proposed to regulate the water levels in the four-tank system can be formulated as follows:

$$\min_{\mathbf{u}(k)\in\mathbb{R}^{m\times H_p}} \quad J(x_0, \mathbf{u}(k)), \quad (8)$$

subject to

- system model (2) over H_p,
- state constraints (3) over H_p,
- input constraints (4) over H_p,

It is useful to relax the state constraints, this leads to reduced computation time while it keeps the states in the admissible range of operation. Additional costs could be added to represent this relaxation of constraints but experiments showed it was not necessary to add this additional weights to the cost function.

To define the control horizon (H_p) it is necessary to take into account the sampling time and the dynamics of the system. If H_p is too small the controller does not produce a significant impact on the system due to the small optimization window, thus the cost of the control action is too big compared to the final effect. In consequence, H_p has to be at least big enough to be able to see significant results in the relevant outputs, corresponding to states x_1 and x_2. A critical issue in the election of H_p is the stability of the closed loop system, the values considered for H_p have never presented stability problems. Finally the control horizon (H_c) must be defined, the computation time is not critical and allows to use H_c equal to H_p. In table 1 the parameter configuration for the simulations are presented.

4 Simulation Results

The initial state for all simulations is $x_0 = (h_1, h_2, h_3, h_4) = (0.5955, 0.6616, 0.5384, 0.7682)$ in [m]. The simulations have been carried out using `fmincon` in MATLAB® R2011a (32 bits), running in a PC Intel® Core^{TM} i7-3770 at 3.40GHz with 8GB of RAM.

4.1 Controller Setup

One of the advantages of NMPC algorithms is the high degree of configuration that they offer (control and prediction horizons, penalization terms, etc.). Table 1 shows the controller setup parameters and the computational effort for the simulation scenario proposed for the CEA competition.

Table 1: NMPC setup parameters.

Parameter	Variable	Value
Prediction horizon	H_p	120
Control horizon	H_c	120
Sampling time	T_m	5 s
Simulation time	T_{sim}	4800 s
Computing time	CPU_t	10 min
Av. optimization time (figure 9)	T_{opt}	0.58 s

In figure 2, a general representation of the closed-loop control scheme for the NMPC approach implementation is showed.

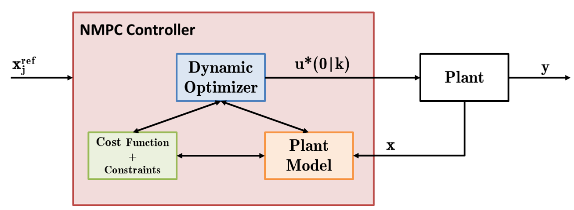

Figure 2: Closed-loop control scheme

4.2 Results and Discussion

The computing time for each of the simulations is around 10 minutes each time the algorithm runs to study 80 minutes of the behaviour of the system. The computing time is below the real response of the system, hence the adapted solution is adequate to be implemented in a real plant.

The behaviour of the water level of each tank versus the ideal trajectory planned by the reference generator is represented in figures 3, 4, 5 and 6. As it can be seen in previous figures, not always the real tank level arrives to the ideal trajectory with the same velocity. The prioritisation of the minimal cost of the trajectory over the reference tracking of the tank levels result in the shown water dynamical behaviour.

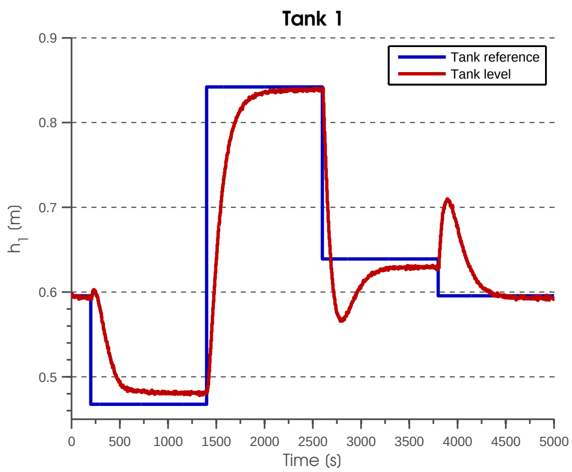

Figure 3: Tank 1 level

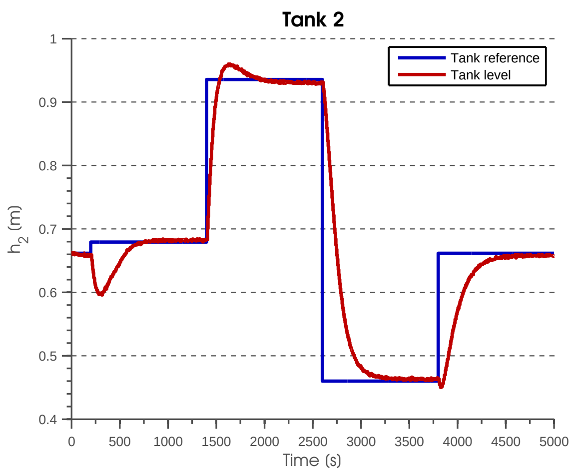

Figure 4: Tank 2 level

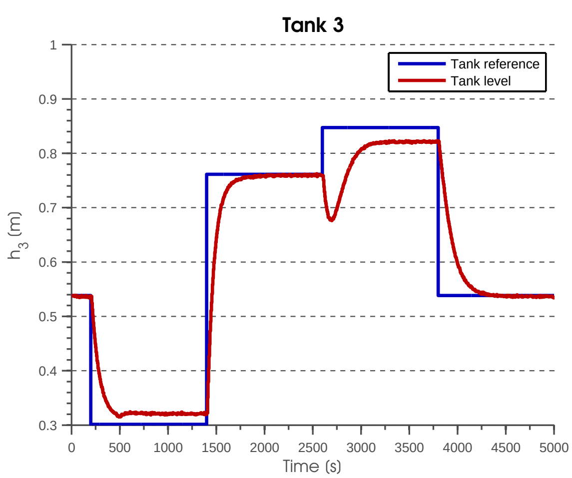

Figure 5: Tank 3 level

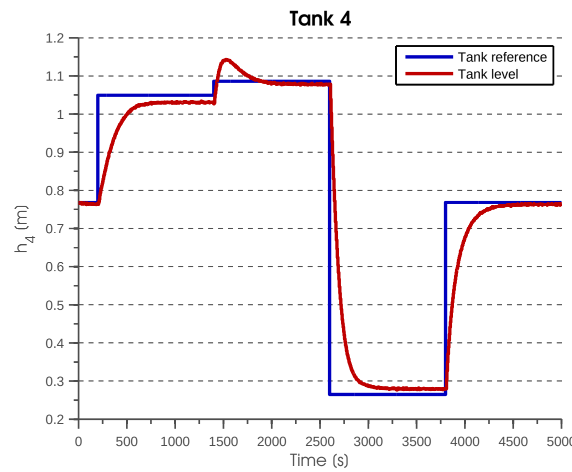

Figure 6: Tank 4 level

Water flows can also be represented. In figure 7, the behaviour of both q_a and q_b versus the ideal trajectories q_a^* and q_b^* are shown.

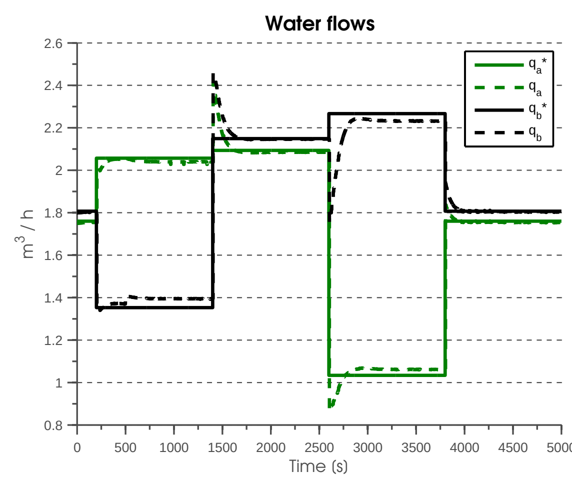

Figure 7: Water flows behaviour

The dynamical behaviour of the cost function is shown in figure 8 where it can be seen that at each change of the ideal trajectories, there is a variation of the cost. The cost function is returned to zero approximately in 300-400 seconds after the trajectory variation. Figure 8 shows the computation time at each simulation step during the case study.

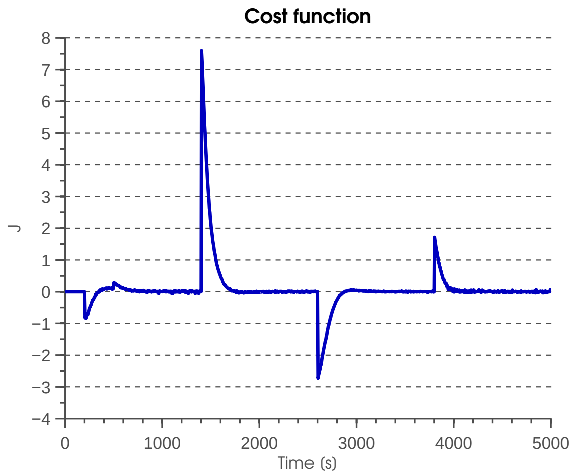

Figure 8: NMPC cost function

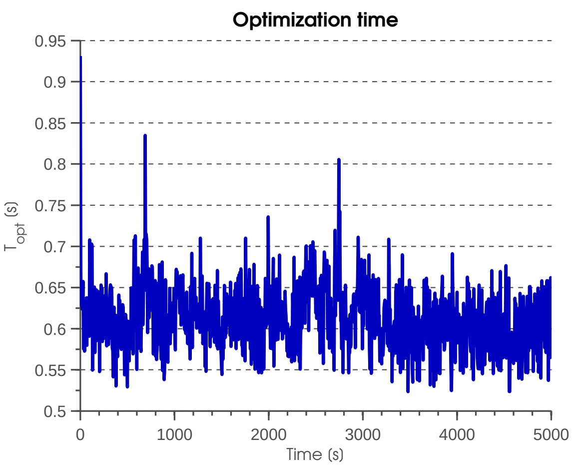

Figure 9: Optimization time at each simulation step

Finally, the representation of the ID performance index over time is included in figure 10. After the simulation is completed, a final ID value of 431.2 is obtained.

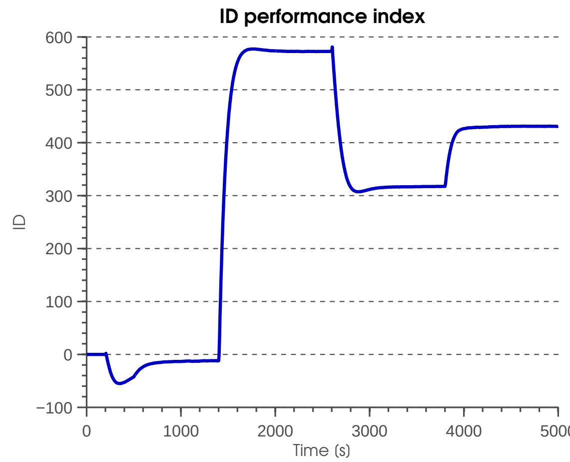

Figure 10: ID performance index value

5 Conclusions

A NMPC controller has been designed and implemented to be applied to the four-tanks system. The performance of the controller has been evaluated, obtaining low ID values in the simulation scenario proposed in the contest. The proposed controller minimizes a cost function that takes into account two different control objectives: the minimization of the energetic consumption of the plant and the maximization of the accumulated water volume in the lower deposits.

To implement the controller in the real plant, further tuning will have to be performed in situ to adjust the controller parameters in order to behave adequately taking into account real operation parameters and the possible disturbances that may appear. Moreover, the proposed optimizer function `fmincon` can be substituted and/or modified to improve the computation time by means of optimization methods that are more appropriate for the nonlinearities present in the studied case study.

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